Hint and Solutions

Chemistry [PART-I]

1. **B**

I. $2H_2O + 2e^- \longrightarrow H_2(g) + 2OH^-,\ E^o_{red} = E^o_1 = ?$

II. $H_2(g) \longrightarrow 2H^+ + 2e^-,\quad E^o_{red} = E^o_2 = 0.0V$

III. $2H_2O \rightleftharpoons 2H^+ + 2OH^-,\ E^o_3 = ?$

Now, $\Delta G^o_3 = \Delta G^o_1 + \Delta G^o_2$

$\Rightarrow -2.303RT\log(K_w)^2 = -2 \times F \times E^o_1 - 2 \times F \times E^o_2$

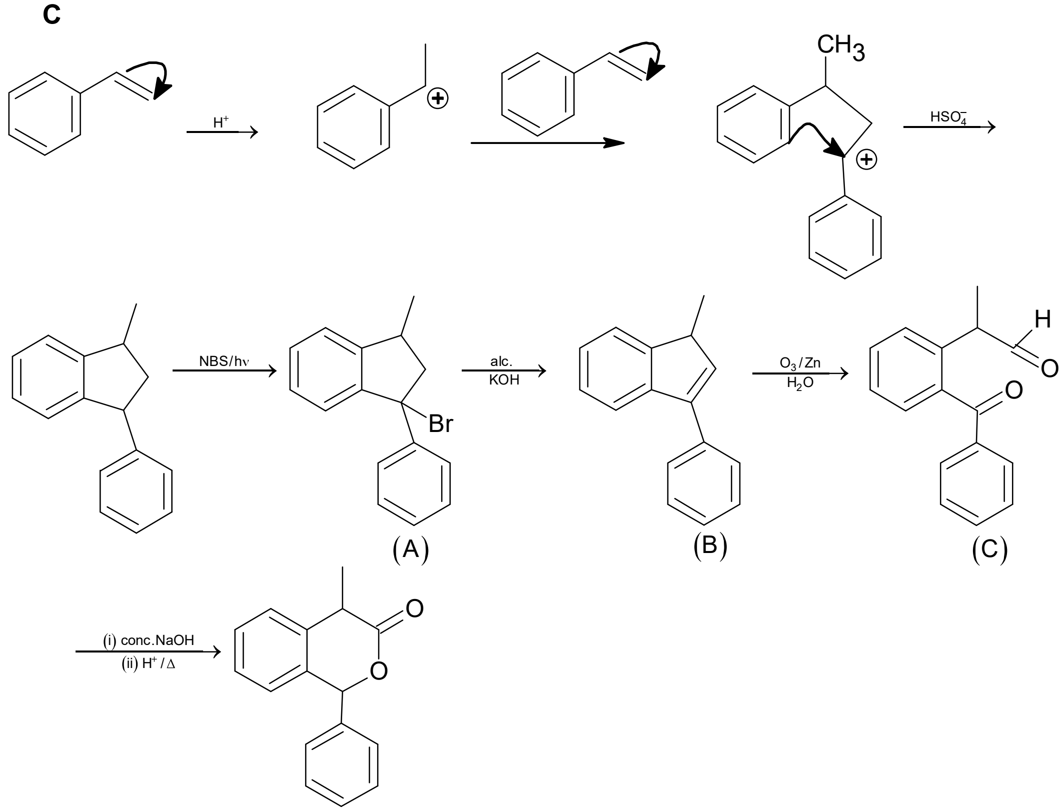
$\therefore E^o_1 = -\ 0.8274$ V

Now, $E_{red} = E^o_{red} - \dfrac{0.0591}{2}\log\left[OH^-\right]^2 \times p_{H_2}$

$= -\ 0.8274 - \dfrac{0.0591}{2}\log\left(10^{-4}\right)^2 \times 1$

$= -\ 0.5910$ V

2. **C**



3. **A**

Let radius of hollow sphere B =r

$\therefore$ Edge length a $= \dfrac{4r}{\sqrt{3}}$

Volume of unit cell = $a^3 = \left(\dfrac{4r}{\sqrt{3}}\right)^3$

Volume of B unoccupied by A (having radius r/2) in unit cell =

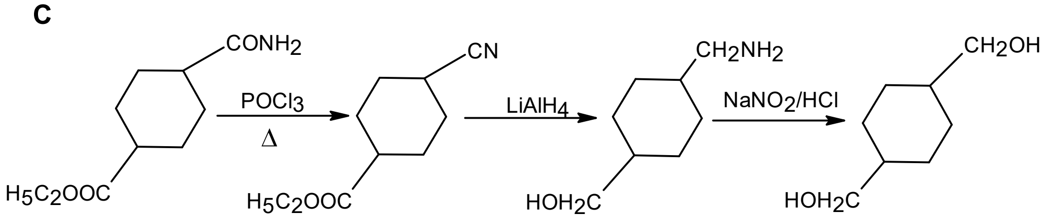
$$= \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 \right) \times 2$$

$$\therefore \frac{\text{volume of B unoccupied by A in unit cell}}{\text{volume of unit cell}} = \frac{\frac{4}{3}\pi \frac{7r^3}{8} \times 2}{\left(\frac{4r}{\sqrt{3}}\right)^3} = \frac{7\pi\sqrt{3}}{64}$$

4. **B**

Since BF_3 is not finally regenerated it is not used as a catalyst and BF_3 is an electrophile not a nucleophile.

5. **C**



6. **A**

7. **A**

$\alpha = \frac{\Lambda^C}{\Lambda^0}$; $\Lambda^C = \frac{1000k}{C}$ =40 and van't hoff factor, i=1+α for a solute like acetic acid (AB type)

8. **B**

In the second reaction which is an example of Williamson's synthesis elimination dominates giving alkene as the major product.

9. **A, B, C, D**

Calculation of mol. wt. of BOH:

eq. of the base = eq. of H_2SO_4 used

$\frac{0.496}{m_B} \times 1 = 40 \times 10^{-3} \times \frac{1}{2} \times 2 \Rightarrow m_B = 1.24 \text{ gmol}^{-1}$

Now:

$\Delta T_f = i \times K_f \times m$

$0.165 = i \times 1.86 \times \frac{1.5}{12.4} \times \frac{1000}{150}$

$i = \frac{0.165 \times 12.4}{1.86} = 1 - 1 = 1 + \alpha \therefore \alpha = 0.1$

So, $\left(OH^-\right) = C\alpha = 0.8 \times 0.1 = 8 \times 10^{-2}$ $\therefore pOH = 1.1$ $\therefore pH = 12.9$

So, $K_b = C\alpha^2 = 0.8 \times (0.1)^2 = 8 \times 10^{-3}$

Also, $\pi = iCRT$

= 1.1 × 0.8 × 0.0821 × 300

= 21.67 atm

10. **A, B, C**

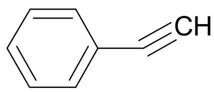
11. **A, B, C, D**

12. **A, B, D**

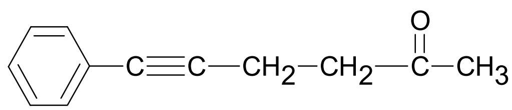
13. **B**

14. **C**

15. **D**



(A)



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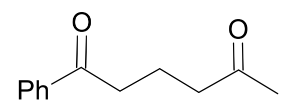
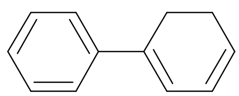


(C)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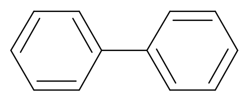
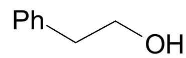

(E)



(F)

16. **C**

$$\underset{}{NH_4HS(s)} \rightleftharpoons \underset{27\times10^{-2}}{NH_3(g)} + \underset{x}{H_2S(g)}; \qquad \underset{3a}{2NH_3(g)} \rightleftharpoons \underset{\frac{1}{3}a}{N_2(g)} + \underset{a}{3H_2(g)}$$

$Kc = 27\times10^{-2}\,x = 8.91\times10^{-2}$

$x = \dfrac{8.91\times10^{-2}}{2.7\times10^{-1}}$

$= 3.3\times10^{-1}$

No of mol of $NH_3 = 0.27$

$H_2S = 0.33$

$N_2 = 0.03$

$H_2 = 0.09$

$x_{H_2} = \dfrac{0.09}{0.72} = \dfrac{1}{8}$

$Kc = \dfrac{(a/3)a^3}{(3a)^2} = \dfrac{a^4}{3}\times\dfrac{1}{9a^2}$

$\dfrac{a^2}{27} = 3\times10^{-4}$

$\therefore a^2 = 81\times10^{-4}$

$a = 9\times10^{-2}$

17. **C**

Amt of $NH_4HS(s)$ dissociated = amt of $H_2S(g)$ present at equilibrium

$\therefore$ Amount of $NH_4HS(s)$ dissociated = 0.33 mol

$\%(\text{by wt})$ of $NH_4HS(s)$ dissociated $= \dfrac{0.33}{1}\times100 = 33\%$

18. **A**

Weight of the gases present at equilibrium = wt. of NH_4HS (s) dissociated

$\therefore$ weight of the gases present = 0.33 × 51 g

Volume of the gases = 1L

$\therefore$ Density of the gaseous mixtures $= \dfrac{0.33\times51}{1}\,gL^{-1}$ = 16. 83 g L^{-1}

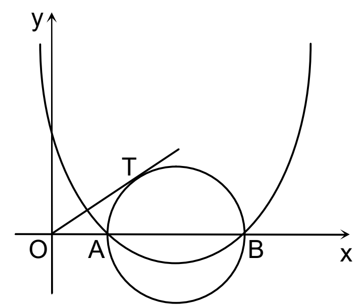
1. **(A) → (p, t); (B) → (s, t); (C) → (r); (D) → (p, q)**

2. **(A) → (t); (B) → (p, t); (C) → (p, r, t); (D) → (q, r, t)**

Mathematics [PART-II]

1. $xy = k^2$
Differentiating the above relation we get xdy + ydx = 0
The orthogonal trajectory to the above differential equation family is xdx – ydy = 0

2. The given curve is a parabola $y = x^2 - \sqrt{5}\,x + 1$
$\Rightarrow A = \left(\frac{\sqrt{5}-1}{2}, 0\right)$, $B = \left(\frac{\sqrt{5}+1}{2}, 0\right)$
$\therefore OT^2 = OA \cdot OB$
$OT^2 = 1$
$\Rightarrow |OT| = 1$



3. $f(x, y) = x^2 + y^2 + 2gx + 2fy + c = 0$
Now, $f(0, \lambda) = \lambda^2 + 2f\lambda + c = 0$ and its roots are (1, 1).
$\Rightarrow f = -1$ and $c = 1$
$f(\lambda, 0) = \lambda^2 + 2g\lambda + c = 0$ and its roots are $\left(\frac{1}{5}, 5\right)$.
$\therefore \frac{1}{5} + 5 = -2g$
$\Rightarrow g = -\frac{13}{5}$ and $\frac{1}{5} \times 5 = c$
Thus, $g = -\frac{13}{5}$, f = –1, c = 1
$r = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{169}{25} + 1 - 1} = \frac{13}{5}$.

5. $\left(\sqrt{(x-3)(x-1)} + 1\right)\log_5 x + \frac{1}{x}\left(\sqrt{2(1-x)(x-3)} + 1\right) \le 0$
If we find the domain of above inequation
then $(x - 3)(x - 1) \ge 0$ also $(1 - x)(x - 3) \ge 0$
$\Rightarrow (-\infty, 1] \cup [3, \infty)$ $(x - 1)(x - 3) \le 0$
$1 \le x \le 3$
Now possible values of x is 1 and 3 but only x = 1 is satisfying the inequality.

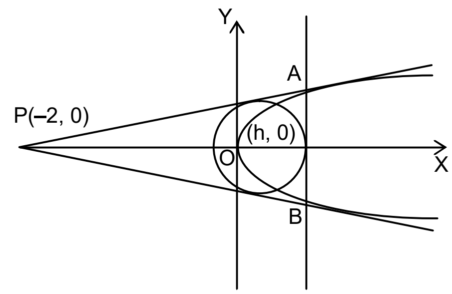
6. $\therefore |x| + |y| = 2$ …..(i)
$|z + 2i| + |z - 2i| = 4$ …..(ii)
eq. (i) represent square & (ii) represent line segment solution are z = ± 2i.

7. f(x) is a decreasing function and for major axis to be x-axis $f(k^2 + 2k + 5) > f(k + 11)$
$\Rightarrow k^2 + 2k + 5 < k + 11$
$\Rightarrow k \in (-3, 2)$.

8. By solving: a + 2b + c = 0, 2a – b + 4c = 0 and a – b + 3c + 4 = 0.

9. $g(x) = \sin\left(\sin^{-1}\sqrt{\{x\}}\right) + \cos\left(\sin^{-1}\sqrt{\{x\}}\right) - 1 = \sqrt{\{x\}} + \cos\left(\cos^{-1}\sqrt{1-\{x\}}\right) - 1 = \sqrt{\{x\}} + \sqrt{1-\{x\}} - 1$
If $x \in I$, then {x} = 0
$\Rightarrow g(x) = 0 \quad \Rightarrow g(x) = g(-x)$
If $x \notin I$, then {–x} = 1 – {x}
$\Rightarrow g(-x) = \sqrt{1-\{x\}} + \sqrt{\{x\}} - 1 = g(x)$.
Hence, g is an even function.

11. Point P clearly lies on the directix of $y^2 = 8x$.
Thus, slopes of PA and PB are 1 and –1 respectively.
Equation of PA : y = x + 2,
Equation of PB : y = –x – 2, Equation of AB : x – 2
Let the centre of the circle be (h, 0) and radius be r.



$\Rightarrow \frac{|h+2|}{\sqrt{2}} = \frac{|h-2|}{1} = r$

$\Rightarrow h^2 + 4 + 4h = 2(h^2 + 4 - 4h)$

$\Rightarrow h^2 - 12h + 4 = 0$

$\therefore h = \frac{12 \pm 8\sqrt{2}}{2} = 6 \pm 4\sqrt{2}$

$\Rightarrow$ radius = $|h - 2| = 4\left(\sqrt{2}-1\right), 4\left(\sqrt{2}+1\right)$.

12. $\det(M - I) = \det(M - I)^T$

$= \det(M^T - I) = \det(-M - I) = \begin{cases} \det(M+I), & \text{if n is even} \\ -\det(M+I), & \text{if n is odd} \end{cases}$

13. 100 – 20 = 80
If we express 80 and 20 on the base 2, 3 and 5 we see that in summation of these two numbers on each base there is a carry.
So, ${}^{100}C_{20}$ is divisible by 2, 3, 5.

14 As in the above question use the base 7 representation.

15. Check divisibility by 2 and 5.

16. $\lim_{x\to 1} \int_2^{f(x)} \frac{2t}{x-1} dt = 4$

$\frac{\lim_{x\to 1} \int_2^{f(x)} 2t dt}{\lim_{x\to 1} x - 1} = 4\left(\frac{0}{0}\text{form}\right)$

L'Hospital rule

$\Rightarrow \frac{\lim_{x\to 1} \frac{d}{dx} \int_2^{f(x)} 2t dt}{1} = 4$

$\Rightarrow \lim_{x\to 1} f(x))^2 f'(x) = 4$

$\Rightarrow f'(1) = 1$.

17. $\frac{\lim_{x\to 0} \int_0^x \frac{t^2 dt}{\sqrt{a+t}}}{\lim_{x\to 0}(x - \sin x)}$ $\left(\frac{0}{0}\text{from}\right)$ ...(1)

L'Hospital rule

$\frac{4a}{2a^{3/2}} = 1$

$\Rightarrow \frac{2}{\sqrt{a}} = 1 \Rightarrow a = 4$.

18. $\lim_{x\to 0}\frac{\int_0^{x^2}\cos t\,dt}{\lim_{x\to 0} x\sin x}$ $\left(\frac{0}{0}\text{form}\right)$

L'Hospital rule

$= \lim_{x\to 0}\frac{\cos^2 x\cdot 2x}{x\cos x+\sin x}$

$= \lim_{x\to 0}\frac{2\left[\cos^2 - \sin x^2\cdot x\cdot 2x\right]}{\cos x - x\sin x+\cos x} = \frac{2}{2} = 1$

SECTION – B

1. (A). $f'(x) < 0 \Rightarrow +x^4e^{-x} - 4x^3e^{-x} < 0$
$x^3e^{-x}(x-4) < 0$.

(B). $c = 0,\ g = -\frac{1}{2}$ as $x^2 + y^2 + 2gx + 2fy + c = 0$ passes through (0, 0), (1, 0) and given touches $x^2 + y^2 = 9$.

(C). If the point (–2a, a + 1) lies in shaded region
$4a^2 + (a+1)^2 - 4 < 0$ and $(a+1)^2 - 4(-2a) < 0$
$\Rightarrow 5a^2 + 2a - 3 < 0$ and $a^2 + 10a + 1 < 0$
$\Rightarrow a \in \left(-1, -5+2\sqrt{6}\right)$.

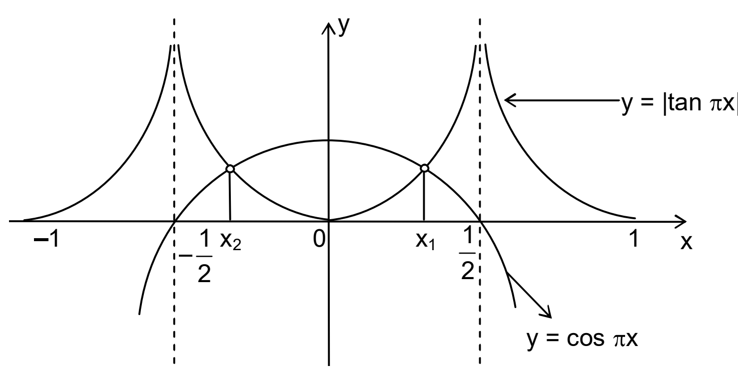
(D). $\frac{\pi}{2} - \sin^{-1}\frac{6x}{1+9x^2} = -\frac{\pi}{2} + 2\tan^{-1}3x$

$\Rightarrow \sin^{-1}\frac{6x}{1+9x^2} = \pi - 2\tan^{-1}3x$

$\Rightarrow \sin^{-1}\left[\frac{2(3x)}{1+(3x)^2}\right] = \pi - 2\tan^{-1}3x$

$\Rightarrow 3x > 1$.

2. (A). Hence at 4 points function is not differentiable



(B). $\because 7! = 2^4 \cdot 3^2 \cdot 5 \cdot 7$ and factor should be of (3n + 1) form and odd, only.
$\therefore$ their sum = 8, $\{\because$ in $2^\alpha 3^\beta 5^\gamma 7^\delta$, $\alpha = 0, \beta = 0, \gamma = 0, \delta = 1\}$.

(C). $\because$ In $2x^2 + 3x + 4 = 0$, roots are pair wise imaginary least of a + b + c = 2 + 3 + 4 = 9.

(D). $f(x) = \int\tan^7 x dx - \log|\cos x| = \int\tan^5 x(\sec^2 x - 1)dx - \log|\cos x|$
$= \frac{\tan^6 x}{6} - \int\tan^5 x\,dx$ ....

Physics [PART-III]

SECTION – A

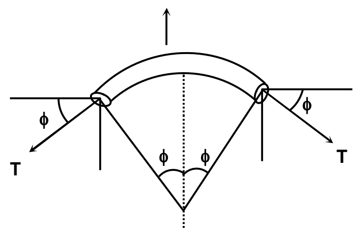
1. $R = 2^{1/3} r$

fractional change = $\frac{\Delta U}{U_i}$

2. $dP = rB\theta$

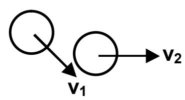
$T\sin\phi = (rB\theta)\pi R^2 \sin^2\phi$

$\sigma = \frac{T}{(2\pi R\sin\phi)t} = \frac{Br\theta R}{2}$



3. Take torque about A

4. After collision



Conservation of momentum in x–direction

$5m \sin 30 = v_1 m \sin 30 + mv_2$

Coefficient of restitution :

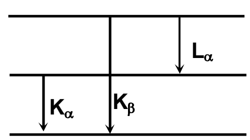
$\frac{1}{2} = \frac{v_2 \cos 60 - v_1}{5}$

5. $F = -\frac{dU}{dr} = \frac{mr^2}{r}$

$r \propto \frac{1}{n^2} \Rightarrow V \propto n^3$

$E \propto n^6$

6. $E_{k\beta} = E_{L\alpha} + E_{K\alpha}$



7. Use Len's maker's formula to find the focal length of the lens in terms of its radius of curvature

$P_{net} = P_{lens} + \underset{\substack{\Downarrow \\ 0}}{P_{plane\ mirror}} + P_{lens}$

8. $\Delta Q = \frac{\Delta\phi}{R}$

9. $f_A - f_B = \pm 2Hz$

$f_B - f_C = \pm 3Hz$

$(f_A - 456) = (456 - f_C)$

10. Maximum compression = $\frac{2mg}{K}$

$U_{spring} = \frac{1}{2}kx^2$

$U_{gravity} = -\,mgx$

$U_{spring} + U_{graw} = -\,\Delta KE$

11. Basic concept of Photo–electric effect

12. Basic concept of gravitation :
    - Shell theorem
    - Gauss Law
    - Gravitational field I intensity
    - Gravitational Potential

13–15

Excess pressure inside the bubble must balance the weight of the mass (when in equilibrium)

$$P_0 + \frac{4T}{R} = P_0 + \frac{Mg}{\pi R^2}$$

$$M = \frac{4\pi TR}{g}$$

$$-\frac{4T}{2R}(\pi R^2) + mg = ma$$

a = g/2

Loss in potential energy of piston = Gain in K.E. of piston + Increase in surface energy

16 – 18

$\vec{B}$ due to long wire carrying current I = $\frac{\mu_0 I}{2\pi r}$

(take care of its directions)

F = qvB sin θ = qvB (θ = 90°)

$$\text{So, } B = \frac{F}{qv} = \frac{3.2\times10^{-20}}{1.6\times10^{-19}\times4\times10^{3}} = 5\times10^{-7}\text{ T}$$

$$\text{Now, } \frac{\mu_0}{2\pi}\left[\frac{I_p}{5}+\frac{I_q}{2}\right] = 5\times10^{-7}$$

$\Rightarrow$ I = 4 amp.

If the distance of point R from third current carrying current is X, then

$B_R = 0$

$$\frac{\mu_0}{4\pi}\frac{2\times2.5}{|x|} + 5\times10^{-7} = 0$$

so x = $\pm$ 1 m

SECTION – B

1. Basic concept of geometrical optics

2. For Case I take torque about COM
   For case II, take torque about any point on the ground.