Hint and Solutions

Chemistry [PART-I]

1. **A**

2. **C**

$$FeC_2O_4 + Fe_2(C_2O_4)_3 \xrightarrow[nf=5]{KMnO_4} Mn^{2+} + Fe^{3+} + CO_2$$

nf = 3 nf = 6

1 $KMnO_4$ moles for FeC_2O_4 oxidation = $2.5 \times \frac{3}{5} = 1.5$

$KMnO_4$ moles for $Fe_2(C_2O_4)$ = $2.5 \times \frac{6}{5} = 3.0$

Total moles of $KMnO_4$ = 4.5

3. **C**

4. **A**

SO_2 is oxidised by MnO_4^- in acidic medium.

$$5SO_2 + 2MnO_4^- \longrightarrow 2Mn^{2+} + 5SO_4^{2-}$$

2mol MnO_4^- ≡ 5 mol SO_2
Hence, 0.4 mol MnO_4^- ≡ 1 mol SO_2

$$2SO_2 + O_2 \rightleftharpoons 2SO_3$$

(2–2x) (1–x) 2x
(2 – 2x) = 1 mol (as determined by MnO_4^-)
∴ x = 0.5

$$\therefore [SO_3] = \frac{2x}{V} = 1M$$

$$[SO_2] = \frac{2-2x}{V} = 0.5M$$

$$[O_2] = \frac{1-x}{V} = 0.5\,M$$

$$\therefore K_c = \frac{[SO_3]^2}{[SO_2]^2[O_2]} = 2$$

5. **A**

6. **A**

7. **A, D**

8. **A, C, D**

9. **A, B, C**

1. **A → (s); B → (s); C → (s); D → (r)**

2. **A → (q, s); B→ (q); C → (r); D → (q, r)**

A → (q), (s)

$Bi^{3+} + H_2O \rightarrow (BiO)^{+} + 2H+$

B→ (q)

$NaAlO_2 + H_2O \rightarrow Al(OH)_3 + NaOH$

C → (r)

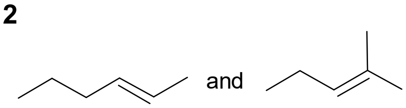
$SiO_4^{4-} + 2H+ \rightarrow Si_2O_7^{6-} + H_2O$

D → (q), (r)

$Na_2B_4O_7 \xrightarrow{HCl} H_3BO_3$

$Na_2B_4O_7 \xrightarrow{H_2O} H_3BO_3$

1. **2**



2. **1**

For the first part, millimole of $Na_2S_2O_3$ consumed = 15 × 0.4 = 6

$I_2 + 2Na_2S_2O_3 \longrightarrow 2NaI + Na_2S_4O_6$ millimoles of I_2 consumed = 3 millimoles (i)

For the second part, the reaction is $3I_2 + 6NaOH \longrightarrow 5NaI + NaIO_3 + 3H_2O$ reacted with I_2

millimoles of NaOH $= (100\times0.3) - (10\times0.3\times2) = 30 - 6 = 24$ millimoles.

Millimoles of I_2 consumed $= \frac{24}{2}$ = 12 millimoles. → (ii)

Total millimoles of I_2 consumed in reaction = (i) + (ii) = 3+ 12 = 15 millimoles

$\therefore$ molarity of $I_2 = \frac{15}{150} = 0.1M$ " 10 times initial molarity of I_2 " $= 0.1\times10 = 1$

Ans = 1

3. **3**

Total weight = 19.6 gm

(a) 2.8 g of molecules, density = 0.75 g /litre

Volume = $\frac{mass}{density} = \frac{2.8}{0.75}$

1 mole $\longrightarrow 22.4$ litres at N.T.P

$\longrightarrow \frac{2.8}{0.75}$ litres at N.T.P

Moles = $\frac{2.8}{0.75} \times \frac{1}{22.4}$

Molecules = $\frac{2.8}{0.75} \times \frac{6}{22.4} \times 10^{23} = 1\times10^{23}$

(b) 11.2 g of molecules, density = 3 g /litre molecules = 1 × 10^{23}

(c) 5.6 g of molecules, density = 1.5 g/ litre molecules = 1 × 10^{23}

Total no of molecules = 3 × 10^{23}

= 10^{-23} N = 3

4. **1**

5. **1**

6. **4**

Pb^{2+}, Cu^{2+}, Cr^{3+}, Zn^{2+}

7. **9**

$CH_3-CH_2-C(CH_3)(H)-CH_2-CH_3$ 3- methyl pentane monochlorination, different products are

$ClH_2-CH_2-C^*(CH_3)(H)-CH_2-CH_3$ chiral, one chiral carbon atom pair of enantiomers

$CH_3-C^*(H)(Cl)-C^*(CH_3)(H)-CH_2-CH_3$ 2 chiral carbon atoms 4 isomers. 2 pairs of enantiomers

4 pairs of diastereomers.

$CH_3-CH_2-C(CH_2Cl)(H)-CH_2-CH_3$ achiral molecule

$CH_3-CH_2-C(CH_3)(Cl)-CH_2-CH_3$ achiral molecule

X = number of pairs of enantiomers = 3
Y = number of pairs of diastereomers = 4
Z = number of isomers which are achiral = 2
Total x + y + z = 9

8. **3**

$\alpha = 0.115 \qquad \alpha = \frac{i-1}{n-1}$

So, i = 1.115

$\Delta T_f = i \times m \times K_f$

$0.416 = 1.115 \times \frac{x \times 1000 \times 1.86}{60 \times 249.25}$

$x = \frac{0.416 \times 60 \times 249.25}{1.115 \times 1000 \times 1.86} = 3$ g

Mathematics [PART-II]
SECTION – A

1. $x_1\, x_2\, x_3\, x_4\, x_5 = 2310 = 3'\, 7'\, 10'\, 11'$ each of 3, 7, 10, 11 can be distributed at 5 places in 5 ways

+ ve integral sols are 5^5

(i) Two are negative and 3 positive then ${}^5C_3 5^5$ ways

(ii) Four are negative and 1 positive then ${}^5C_4 5^5$ ways

(iii) all positive then number of ways is 5^5

2. Any point on (t + 2)(x + y) = 1 is $\left(\alpha,\ \frac{1}{t+2} - \alpha\right)$

Equation of chord of contact is $4x\alpha + 16y\left(\frac{1}{t+2} - \alpha\right) = 1$

$\Rightarrow \alpha(4x - 16y) + \left(\frac{16}{t+2}y - 1\right) = 0$

the chord of contact passes through the intersection of x – 4y = 0 and $\left(\frac{16}{t+2}y - 1\right) = 0$

$\therefore$ the point of intersection is $\left(\frac{t-2}{4}, \frac{t+2}{16}\right)$

It lies inside the ellipse if $4\left(\frac{t-2}{4}\right)^2 + 16\left(\frac{t+2}{16}\right)^2 - 1 < 0$

$\Rightarrow 5t^2 - 12t + 4 < 0 \Rightarrow \frac{2}{5} < t < 2$.

3. n(S) = 5 × 5 × 5 = 125
n(A) = 514, 541, 523, 532, 413, 431, 422, 312, 321, 211
Required probability = $\frac{10}{125} = \frac{2}{25}$.

4. Let the variable chord be x cosα + y sinα = p. Let this chord intersect the hyperbola in A and B
The joint equation of OA and OB is $\frac{x^2}{4} - \frac{y^2}{8} = \left(\frac{x\cos\alpha + y\sin\alpha}{p}\right)^2$

$\Rightarrow \left(\frac{1}{4} - \frac{\cos^2\alpha}{p^2}\right)x^2 - \left(\frac{1}{8} + \frac{\sin^2\alpha}{p^2}\right)y^2 - \frac{2\sin\alpha\cos\alpha}{p^2} = 0$

$\Rightarrow \frac{1}{4} - \frac{\cos^2\alpha}{p^2} - \frac{1}{8} - \frac{\sin^2\alpha}{p^2} = 0 \Rightarrow p^2 = 8$.

The variable line touches the fixed circle, thus perpendicular distance of (0, 0) = Radius
$\therefore$ equation of the circle is $x^2 + y^2 = 8$.

5. $f(x) = [x](x^2 - 25)^n(x^2 + 3)^m(x^2 + 3x + 4)^m$
$f(5) = 0$, $f(5^-) = 4(\text{negative})^n (+)(+)$
but for a local minima at x = 5, $f(5^-) > 0$
$\therefore$ n must be even, m can be any number.

6. $x + y = 2a$, let $z = xy = x(2a - x) = a^2 - (x - a)^2$
z is max if x = a, thus $z_{max} = a^2$
now $P(xy \ge ma^2) = P(x(2a - x) \ge ma^2)$
$\Rightarrow - x^2 + 2ax - ma^2 \ge 0 \Rightarrow x^2 - 2ax + ma^2 \le 0$
for m = $\frac{3}{4}$, P = $\frac{1}{2}$ and for m = $\frac{7}{16}$, P = $\frac{3}{4}$.

7. $\frac{1}{x_{k+1}} = \frac{1}{x_k(x_k+1)} = \frac{1}{x_k} - \frac{1}{x_k+1}$

$\therefore \frac{1}{x_1+1} + \frac{1}{x_2+1} + ... + \frac{1}{x_m+1} = \frac{1}{x_1} - \frac{1}{x_{m+1}}$

also $0 < \frac{1}{x_{m+1}} < 1$

$\therefore \left[\frac{1}{x_1+1} + \frac{1}{x_2+1} + ... + \frac{1}{x_m+1}\right] = 1 \ \forall\, m \in N$.

8. $f'(x) = x^4 + x^3 + 3x^2 + 2kx + 1$

$= \left(x^2 + \frac{x}{2}\right)^2 + \left(3 - \frac{1+k^2}{4}\right)x^2 + \left(\frac{kx}{2} + 1\right)^2 \ge 0$

for this to hold $3 - \frac{1+k^2}{4} \ge 0 \Rightarrow k^2 \le 11$.

9. $\lim\limits_{x\to 0} \sin\left(\frac{\pi\left(1-\cos^m x\right)}{x^n}\right) = \sin\left(\lim\limits_{x\to 0} \frac{\pi(1-\cos^m x)}{x^n}\right) = \sin\left(\lim\limits_{x\to 0} 2\pi \cdot m \cdot \frac{\sin^2 x/2}{x^n}\right)$

$\Rightarrow m \in N$ and n = 1 or 2.

SECTION – B

1. $I(m) = \int_0^{\pi} \ln\left(1 - 2m\cos x + m^2\right)dx$

$I(-m) = \int_0^{\pi} \ln\left(1 + 2m\cos x + m^2\right)dx = \int_0^{\pi} \ln\left(1 + 2m\cos(\pi - x) + m^2\right)dx$

$= \int_0^{\pi} \ln\left(1 - 2m\cos x + m^2\right)dx = I(m)$

$\Rightarrow I(m) = I(-m)$ ...(1)

Also $I(m) + I(-m) = \int_0^{\pi} \ln\left(1 - 2m^2\cos 2x + m^4\right)dx$

Put 2x = t

$\therefore I(m) + I(-m) = I(m^2)$ ...(2)

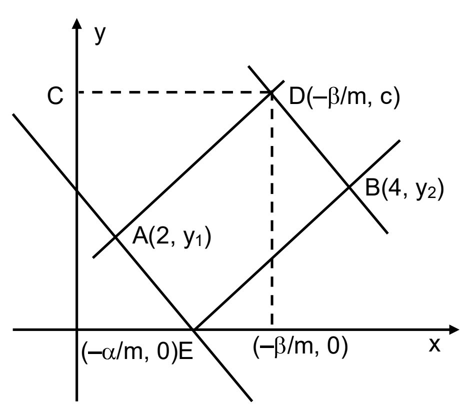
$2I(m) = I(m^2) = \frac{I(m^2)}{I(m)} = 2$.

2. Any prime factor that occurs in the numerator cannot occur in the denominator. There are 4 prime factors of 10!. For each of these prime factors i.e. 2, 3, 5, 7. One must decide only whether it occurs in numerator or denominator. Thus 4 decisions can be made in 2^4 ways. They can be grouped into 8 pairs of reciprocals each containing one factor less than 1. (All other cases are similar to this one)

SECTION – C

1. $y = |mx + \alpha|$ and $y = -|mx + \beta| + c$ intersect in two points whose abcsissae are 2 and 4.
AEBD is a parallelogram.
The diagonal bisect each other

$\therefore -\frac{\alpha}{m} - \frac{\beta}{m} = 2 + 4 \Rightarrow \frac{\alpha+\beta}{m} = -6$.



2. $S = \frac{16 + 3x + 5x}{2} = 8 + 4x$,

Now $3x + 5x > 16 \Rightarrow x > 2$

$3x + 16 > 5x \Rightarrow x < 8 \Rightarrow x \in (2, 8)$

$5x + 16 > 3x \Rightarrow x > -8$
Now $A^2(x) = (8 + 4x)(4x - 18)(x + 8)(8 - x) = (16x^2 - 64)(64 - x^2)$
Let $x^2 = t \Rightarrow t \in (4, 64)$, $f(t) = 16(t - 4)(64 - t) = 16(64t - t^2 - 256 + 4t)$
$\therefore$ $f(t) = -16(t^2 - 68t + 256)$, $f'(t) = 2t - 68 = 0 \Rightarrow t = 34$
$f''(t) = -32 < 0$
Maxima occurs at t = 34
F(34) = 16(30)(30)
$\therefore$ largest possible area = 120.

3. Let x be the $(2009)^{th}$ root of unity $\neq 1$, then

$x^{2009} - 1 = (x-1)(x-w).....(x-w^{2008})$

Taking log on both sides, we get

$\ln(x^{2009}-1) = \ln(x-1) + \ln(x-w) + \ln(x-w^2)..... + \ln(x-w^{2008})$

$\therefore$ On differentiate both the side w.r.t. x, we get

$$\frac{(2009)x^{2008}}{x^{2009}-1} = \frac{1}{x-1} + \sum_{r=1}^{2008}\frac{1}{x-w^r} ....(2)$$

Putting x = 2 in equation (2) , we get

$$\Rightarrow 1 + \sum_{r=1}^{2008}\frac{1}{2-w^r} = \frac{2009(2^{2008})}{2^{2009}-1}$$

Multiplying both sides of above equation by $(2^{2009}-1)$, we get

$$\therefore (2^{2009}-1)\sum_{r=1}^{2008}\frac{1}{2-w^r} = 2009.2^{2008} - 2^{2009} + 1$$

$$= 2^{2008}(2009-2) + 1 = 2^{2008}.2007 + 1 = \left[(a)(2^b) + c\right]$$

$\therefore a = 2007, b = 2008, c = 1$. Hence a + b + c = 4016

4. Without loss of the generality, let a > b > c, then
$a - b \geq 1$, $b - c \geq 1$, $a - c \geq 2$

$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2] \geq 3$

now $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \geq 3(a + b + c)$
Now $(a + b + c)^2 = (a^2 + b^2 + c^2 - ab - bc - ca) + 3(ab + bc + ca) \geq 3 + 3 \times 107$
$a + b + c \geq 18$
$\therefore$ $a^3 + b^3 + c^3 - 3abc \geq 3 \times 18 = 54$.

5. Let Q(tanC, cotC), then

$\overrightarrow{PQ} = (\tan C - \sin A)\hat{i} + (\cot C - \cot B)\hat{j}$

$|\overrightarrow{PQ}|^2 = (\tan C - \sin A)^2 + (\cot C - \cot B)^2$

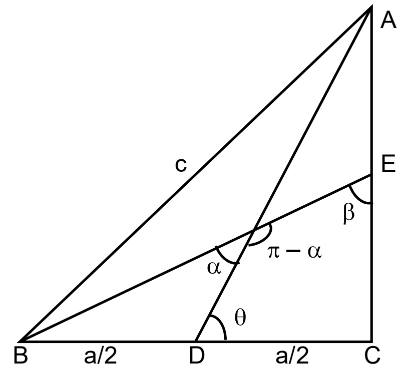
$|\overrightarrow{PQ}| = \overline{OQ} - \overline{OP}$

$|\overrightarrow{PQ}|^2 = \sqrt{\tan^2 C + \cot^2 C} - 1$

$= = \left(\sqrt{(\tan^2 C - \cot^2 C)^2 + 2} - 1\right)^2$

min(PQ) = $(\sqrt{2}-1)^2 = 3 - 2\sqrt{2}$.

6. Let α be the acute angle between AD and BE.



Then $\tan\alpha = \left|\frac{2-1}{1+2}\right| = \frac{1}{3}$.

Now $\theta + 180° - \alpha + \beta + 90° = 360°$

$\alpha = \theta + \beta - 90°$

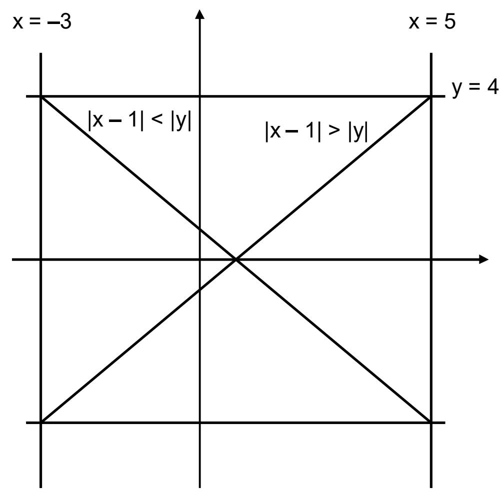
$\tan\alpha = -\tan(\theta + \beta)$

$\Rightarrow -3 = \frac{\tan\theta + \tan\beta}{1-\tan\theta\tan\beta} = \frac{\frac{2b}{a}+\frac{2a}{b}}{1-\frac{2b}{a}\cdot\frac{2a}{b}} = \frac{2b^2+2a^2}{-3ab}$

$\Rightarrow 9ab = 2c^2$.

7. $f(-2) < 0$, $f(-1) > 0$, $f(0) > 0$, $f(1) < 0$, $f(2) > 0$.
Thus $-2 < \alpha < -1$, $0 < \beta < 1$, $1 < \gamma < 2 \Rightarrow [\alpha] = -2$, $[\beta] = 0$, $[\gamma] = 1$
Hence the required quantity is 5.

8. Required area = $8 \times 8 = 64$.



Physics [PART-III]

1. $Q = \Delta U + W$

   $C = C_V + \frac{P}{n}\frac{dV}{dT}$

   $\Rightarrow C = C_V - \frac{R}{\alpha V}(1 - \alpha V)$

2. in a resonance $X_C = X_L$ and $I_e = \frac{V}{R}$

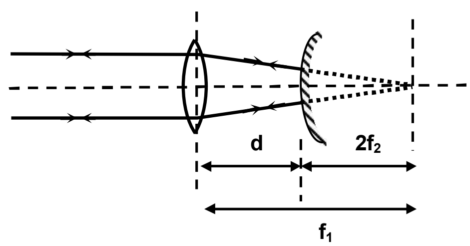
   $\Rightarrow R = 10\ \Omega$

   $\therefore$ in case of d. c. source $i = \frac{6}{10+2}$

3. $v_{avg} \propto \left(\frac{RT}{M}\right)^{1/2}$

   $\lambda \propto \frac{1}{p}$

4.



5. Basics of ray optics

6. Bases of induced Electric field

7. Current in the branch DH is 0

8. $\lambda = \lambda_\alpha + \lambda_\beta$

   $\lambda = \frac{0.693}{T_{1/2}}$

   $\lambda_\beta = \frac{66}{100}\lambda$, $\lambda_\alpha = \frac{34}{100}\lambda$

9. Capacitor basics

SECTION –C

1. Find velocity just before collision (i.e. at the height of 3m) and just after collision. After collision, the particle will take a parabolic path.

2. Use the formula :

   $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

3. use the formula

$$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

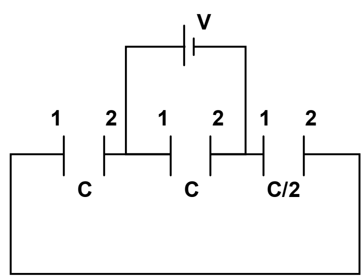
4. At O, the inference is constructive.
So, intensity = 4 I_0

5. $$\mu_{medium} = \frac{\lambda_{vacuum}}{\lambda_{medium}} = \frac{n\lambda_{vacuum}}{n\lambda_{medium}} = \frac{2}{1}$$

6. $$\frac{dN_2}{dt} = \lambda N_1 - 2\lambda N_2 = 0$$
$$\frac{N_1}{N_2} = 2$$

7. Since, potential drop is not changing, E should also not change

8.



solve the circuit to get result.