Chemistry

PART – I

SECTION – A

1. **B**

$$\frac{r_X}{r_{CH_4}} = \sqrt{\frac{M_{CH_4}}{M_X}}$$

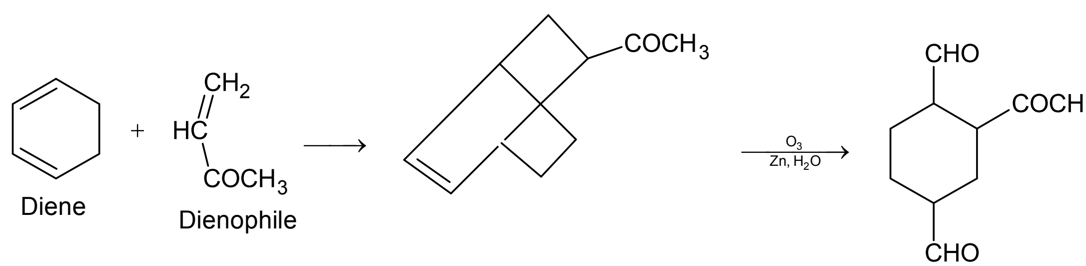
$$\left(\frac{2}{3}\right)^2 = \frac{M_{CH_4}}{M_X}$$

$$M_X = 16 \times \frac{9}{4} = 36$$

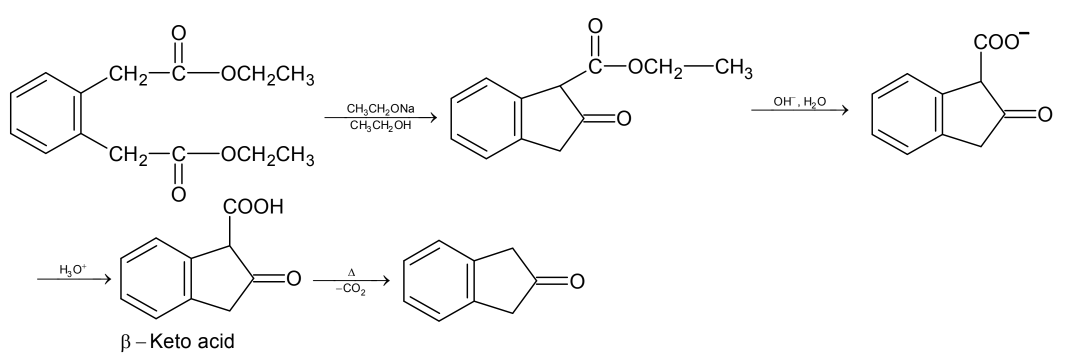
$$V_m = \frac{36 \times 1000}{800} = 45\text{ L}$$

$$Z = \frac{PV_m}{RT} = \frac{1 \times 45}{0.0821 \times 400} = 1.37$$

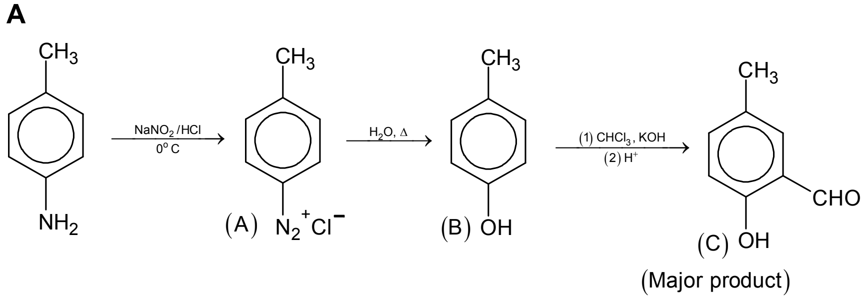
2. **B**



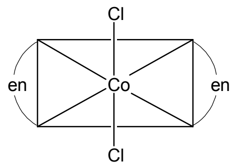
3. **B**



4. **A**



5. **D**



Optically inactive due to plane of symmetry.

6. **B**

$HIO_4 < HBrO_4 < HClO_4$ (correct acidic strength)

7. **B**

$$2XeF_2 + 2H_2O \longrightarrow 2Xe + 4HF + O_2$$
$$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$$
$$XeF_6 + H_2O \longrightarrow XeOF_4 + 2HF$$
$$XeO_3 + 2XeF_6 \longrightarrow 3XeOF_4$$

8. **A**

Wolff Kishner reduction of carbonyl compounds produces hydrocarbon.

9. **B**

Temperature remains same
$q_1 = m_1sT_1$
$q_2 = m_2sT_2$
since, $m_1 = 2m_2$ and $q_1 = 2q_2$, $T_1 = T_2$.

10. **A**

$$\underset{0.1-x}{Sb_2S_3(s)} + \underset{0.1-3x}{3H_2(g)} \rightleftharpoons \underset{2x}{2Sb(s)} + \underset{3x}{3H_2S(g)}$$
$$H_2S + Pb^{2+} \longrightarrow PbS + 2H^+$$

Number of mole of PbS = number of mole of H_2S = $\frac{11.9}{238} = 0.05$

$\therefore 3x = 0.05$

At equilibrium $[H_2]$ = $\frac{0.1-0.05}{2} = \frac{0.05}{2}$

$$[H_2S] = \frac{0.05}{2}$$

$$K_P = \frac{[H_2S]^3}{[H_2]^3} = \frac{(0.05/2)^3}{(0.05/2)^3} = 1$$

11. **A**

$K_P = 10^{-4}\ atm^2$

$\Delta G^o = -2.303\ RT\log K$
$= -2.303 \times 8.314 \times 298 \times \log 10^{-4}$
$= 22.8$ kJ

12. **B**

$\Delta G^o = \Delta H^o - T\Delta S^o$

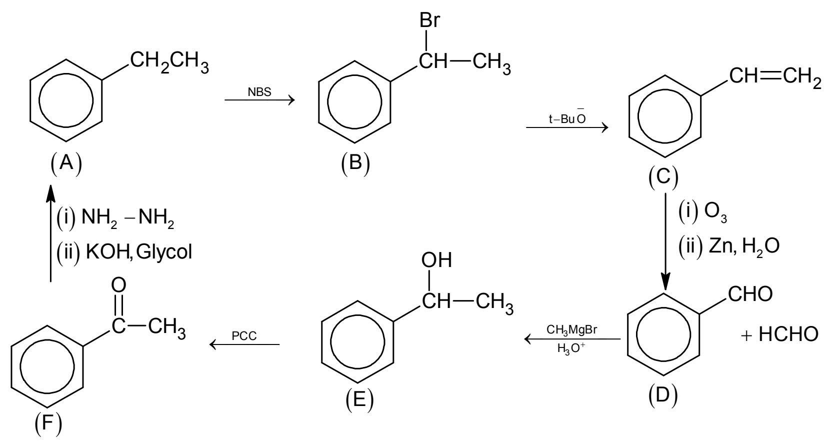
$\Delta S^o = \dfrac{\Delta H^o - \Delta G^o}{T}$

$= \dfrac{100 - 22.8}{298}$

= 0.259 KJ K^{-1} mol^{-1}
= 259 J K^{-1} mol^{-1}

13. **B**

14. **B**



15. **B**

16. **D**

Solution for the Q. No. 15 & 16

$CaCO_3(s) \xrightarrow{\Delta} CaO(s) + CO_2(g)$
(P) (Q) (R)

$CaO + 3C(s) \xrightarrow{\Delta} CaC_2(s) + CO(g)$
(Q) (Carbon) (S) (T)

$CO(g) + \frac{1}{2}O_2 \xrightarrow{\Delta} CO_2(g)$
(T) (R)

$CaC_2 + N_2(g) \xrightarrow{\Delta} CaCN_2 + C(s)$
(S) (U) Carbon

$CaCN_2 + CO_2 + H_2O \longrightarrow H_2NCN + CaCO_3$
(U) (V) (P)

17. **B**

18. **C**

$[NiCl_4]^{2-}$	Tetrahedral,	Paramagnetic
$[Ni(CN)_4]^{2-}$	Square planar,	Diamagnetic
$[Ni(CO)_4]$	Tetrahedral,	Diamagnetic

$[Cu(NH_3)_4]^{2+}$ Square planar, Paramagnetic

19. **D**
(P) Baeyer-Villiger oxidation
(Q) Beckmann rearrangement
(R) Birch reduction
(S) Carbylamine reaction

20. **B**

Mathematics PART – II

SECTION – A

1. $Z_1 = \cos\alpha + i\sin\alpha, \quad Z_2 = \cos\beta + i\sin\beta$

$$|Z_1 + Z_2| + |Z_1 - Z_2| = \sqrt{2 + 2\cos(\alpha - \beta)} + \sqrt{2 - 2\cos(\alpha - \beta)}$$

$let\, \alpha - \beta = \theta$

$$2\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2} = 2\sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

3. Let $\sqrt{x} = \cos\theta$, $\theta = \cos^{-1}\sqrt{x}$, $x = \cos^2\theta$
$dx = -2\cos\theta\sin\theta\, d\theta$

$$I = \int\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} \times \frac{-2\cos\theta\sin\theta d\theta}{\cos^2\theta} = \int\frac{2\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} \times \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\theta} d\theta$$

$$= -4\int\frac{\cos^2\frac{\theta}{2}}{\cos\theta} d\theta = -4\int\frac{1+\cos\theta}{2\cos\theta} d\theta$$

$$= -2\int(\sec\theta + 1)\, d\theta = -2\left[\cos^{-1}\sqrt{x} + \log\left|\left(\frac{1}{\sqrt{x}} + \frac{\sqrt{1-x}}{\sqrt{x}}\right)\right|\right] + c$$

4. $m_1 = \tan\theta_1 \Rightarrow m_2 = \tan\theta_2$ and $m_3 = \tan\theta_3$
Where m_1, m_2 and m_3 are slopes of ℓ_1, ℓ_2 and ℓ_3 respectively
$k = mh - 2am - am^3$
$am^3 + (2a - h)m + k = 0$
$m_1 + m_2 + m_3 = 0$

$$m_1m_2 + m_2m_3 + m_3m_1 = \left(\frac{2a-h}{a}\right)$$

$$m_1m_2m_3 = \frac{-k}{a}$$

$\tan(\theta_1 + \theta_2 + \theta_3) = \tan\alpha$

$$\frac{m_1 + m_2 + m_3 - m_1m_2m_3}{1 - m_1m_2 - m_2m_3 - m_3m_1} = \tan\alpha$$

$$\frac{0+\left(\frac{k}{a}\right)}{1-\left(\frac{2a-h}{a}\right)} = \tan\alpha$$

$\Rightarrow$ y = tan α (x – a) straight line

5. $\frac{\pi}{x} = t$ as $x \to \infty$, $t \to 0$

$$L = \lim_{t\to 0} \frac{\pi^2}{t^2}\sin\left(\ln\sqrt{\cos t}\right)$$

$$= \pi^2 \lim_{t\to 0} \frac{\sin\left[\frac{\ln(\cos t)}{2}\right]}{\left[\frac{\ln(\cos t)}{2}\right]} \cdot \frac{\ln(\cos t)}{2t^2} = \frac{\pi^2}{2}\lim_{t\to 0}\frac{\ln\cos t}{t^2}$$

$$= \frac{\pi^2}{2}\lim_{t\to 0}\left(-\frac{\tan t}{2t}\right) \quad \text{(L Hospital Rule)}$$

$$= -\frac{\pi^2}{4}$$

6. $$\lim_{x\to\infty}\left(\frac{x^2(x+1)}{x^2+2\cos x} + \frac{x+\sin x}{x^2+2\cos x} - a\sin x - bx + c\right) = 4$$

$$\Rightarrow \lim_{x\to\infty}\left(x+1+0-a\sin x - bx + c\right) = 4$$

$$\Rightarrow \lim_{x\to\infty}\left((1-b)x + c + 1 - a\sin x\right) = 4$$

$\Rightarrow$ b = 1, c + 1 = 4, a = 0

7. $f(x) = x^3 + 2x + \cos x + \tan x$
$f'(x) = 3x^2 + 2 - \sin x + \sec^2 x$
$f'(x) = 3x^2 + (2 - \sin x) + \sec^2 x > 0$
Hence, f(x) is increasing function
$\therefore$ f(0) = 1 and f(–1) < 0
Hence, one solution in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

8. $y = ax^3 + bx^2 + cx + 5$
Now s …(i)
and $\left.\frac{dy}{dx}\right|_{(0,5)} = 3 \Rightarrow c = 3$ …(ii)
Curve passes through (–2, 0) we get
$-8a + 4b - 2c = -5$ …(iii)
$\Rightarrow a = -\frac{1}{2}, b = \frac{-3}{4}, c = 3$ (from (i), (ii), (iii))

9. Take $t = xe^x$

$$I = \int_0^e \frac{\sin^2(t)\,dt}{\sin^2(t) + \sin^2(e-t)}$$

$$I = \int_0^e \frac{\sin^2(e-t)dt}{\sin^2(e-t)+\sin^2(t)}$$

$$2I = \int_0^e 1\,dt$$

$$I = \frac{e}{2}$$

10. $$L = \lim_{x\to 0} \frac{x^2 \int_0^x \left(\sqrt[100]{p(t)+1}-1\right)dt}{x^4}$$

$$L = \lim_{x\to 0} \frac{\sqrt[100]{p(x)+1}-1}{2x}$$ (Using Libinitz rule)

$$\Rightarrow L = \frac{1}{200}$$

11. 2 lies between the roots $\Rightarrow f(2) < 0 \Rightarrow 4 - 2(m - 3) + m < 0 \Rightarrow m > 10$

12. $f(2) > 0$, $D \ge 0$, $\frac{-b}{2a} > 2 \Rightarrow m < 10$ and $m \in (-\infty, 1] \cup [9, \infty)$ and $m - 3 > 4 \Rightarrow m \in [9, 10)$

13. $$\underset{x\to\alpha}{\text{Lt}} \frac{1-\cos(ax^3+bx^2+cx+d)}{(ax^3+bx^2+cx+d)^2} \times \frac{a^2(x-\alpha)^2(x-\beta)^2(x-\gamma)^2}{(x-\alpha)^2} = \frac{1}{2}a^2(\alpha-\beta)^2(\alpha-\gamma)^2$$

14. $$\lim_{x\to 0} \frac{1-\cos(1-\cos x)}{x^4} = \lim_{x\to 0} \frac{1-\cos(1-\cos x)}{(1-\cos x)^2} \times \left(\frac{1-\cos x}{x^2}\right)^2 = \frac{1}{2}\times\frac{1}{4} = \frac{1}{8}$$

15. If the given plane intersect in a point then $\Delta_4 \ne 0$ i.e.,
$\lambda \ne 4, -3$
$\therefore \lambda \ne 4$.

16. If the given planes from a triangular prism, then $\Delta_4 = 0$ and none of Δ_1 , Δ_2 , Δ_3 is zero for $\lambda = 4$, the given planes a triangular prism.

17. (P) and (Q)

$S_n = 1! + 2! + 3! \ldots\ldots 7! + 8\,I$ (where I is an integer)

$S_n = 4340 + 8\,I$

Also, $8\left[\frac{S_n}{8}\right] = 4336 + 8I$

$$S_n - 8\left[\frac{S_n}{8}\right] = 4$$

(R) $$L = \lim_{n\to\infty}\left(\frac{n^2}{n^2-1}\right)^{\tan\left(\frac{1}{\sqrt{n}}\right)}$$

$$\ln L = \lim_{n\to\infty} \tan\left(\frac{1}{\sqrt{n}}\right)\ln\left(\frac{n^2}{n^2-1}\right)$$

$$\ln L = \lim_{n\to\infty} \frac{\ln\left(\frac{n^2}{n^2-1}\right)}{\cot\left(\frac{1}{\sqrt{n}}\right)}$$

Now use L'hospital rule and after solving, we get L = 1

(S) $$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{\ln \frac{2i}{n}}{n} \right) = \int_0^1 \ln 2x dx = \left[x\ln 2 + x\ln x - x \right]_0^1 = \ln \frac{2}{e}$$

18. Use basic probability

19. (P) ax = 2nπ and bx = 2mπ both should hold simultaneously for solution of equation

$\Rightarrow \frac{a}{b} = \frac{n}{m}$ (hence $\frac{a}{b}$ must be rational)

or LCM of a, b must exist. Also x = 0 is one solution

(Q) LCM of $\left(1, \sqrt{2}\right)$ does not exist or $\frac{1}{\sqrt{2}}$ is irrational number

(R) Using graph, we can say equation has infininte solutions

(S) For solution to exist x, πx should be odd multiple of π together i.e. x = (2n + 1)π and πx = (2m + 1)π

$\frac{1}{\pi} = \frac{2n+1}{2m+1}$ which is not true, hence no solution

20. (P) $z = x + iy;$

$$c = \frac{x}{x^2+y^2}; \quad x^2 + y^2 - \frac{x}{c} = 0; \ (x, y) \neq (0, 0)$$

(Q) $|z-2| = |z-i|$

$A(2,0), \ B(0,1)$. Locus is perpendicular bisector of AB.

(R) $S_1(-1, 0), S_2(0, 3). \ P(x, y)$; $S_1S_2 = \sqrt{10} < 4.$

$PS_1 + PS_2 = 4$

(S) $(z - \bar{z})^2 + 8(z + \bar{z}) = 0; \ z = x + iy$

$4y^2 = 16x; \ y^2 = 4x$

Physics

PART – III

SECTION – A

1. Considering a small spherical volume of radius r and thickness dr and applying Gauss' Law

$$(E + dE)\, 4\pi\, (r + dr)^2 - E \times 4\pi r^2 = \frac{\rho 4\pi r^2 dr}{\varepsilon_0}$$

$\Rightarrow \quad (E + dE)(r^2 + 2rdr) - E \times r^2 = \frac{\rho r^2 dr}{\varepsilon_0}$ (Neglecting the smaller terms)

$\Rightarrow \quad 2E\, rdr + dEr^2 = \frac{\rho r^2 dr}{\varepsilon_0} \Rightarrow 2(kr^a)rdr + kar^{a+1}dr = \frac{\rho r^2 dr}{\varepsilon_0}$

$\Rightarrow \rho \propto r^{a-1}$

2. Frequency of wave reaching the moving object

$$f_1 = \frac{v+u}{v} f$$

Frequency of wave reaching the transmitter after reflection from the moving object

$$f_2 = \frac{v}{v-u} f_1 = \left(\frac{v}{v-u}\right)\left(\frac{v}{v+u}\right) f = \left(\frac{v+u}{v-u}\right) f$$

Beat frequency $f_2 - f_f = \frac{2u}{v-u} f$

3. $f = \frac{6v}{2L} = \frac{3v}{L}$

$$\lambda = \frac{v}{f} = \frac{L}{3} \Rightarrow L = 3\lambda$$

$$\Delta p_0 = (\Delta p_0)_{max} \sin kx$$

$$\Rightarrow \sin kx = \frac{1}{\sqrt{2}}$$

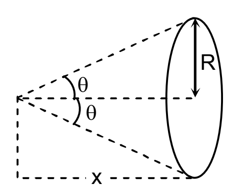
$$\Rightarrow kx = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{4 \times \frac{2\pi}{\lambda}} = \frac{\lambda}{8}$$

$$\Rightarrow \frac{\lambda}{8} = \frac{40}{2} = 20\text{cm}$$

$\Rightarrow$ L = 160 cm

$\Rightarrow$ L = 3λ = 4.8 m

4. $\phi = \frac{q}{2\varepsilon_0}(1-\cos\theta)$



$$= \frac{q}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2+x^2}}\right)$$

$$F = qE = q\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2+x^2}}\right)$$

$\Rightarrow$ F = σφ

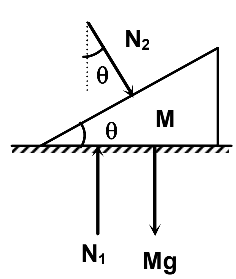
5. $f \le \mu N_1$

$N_2 \sin\theta \le \mu(Mg + N_2\cos\theta)$

$N_2 \sin\theta \le \mu(Mg + mg\cos^2\theta)$

$\Rightarrow mg\cos\theta\sin\theta \le \mu(Mg + mg\cos^2\theta)$

$$\Rightarrow \mu \ge \frac{m\cos\theta\sin\theta}{M + m\cos^2\theta}$$



6. From conservation of energy

$$\frac{1}{2} I_{cm}\omega^2 = pE = qE\ell$$

$$\Rightarrow \frac{1}{2}\frac{2m\ell^2}{3}\omega^2 = qE\ell$$

$\Rightarrow \quad \omega = \sqrt{\frac{3qE}{m\ell}}$

Velocity of negatively charged particle

$v = \omega \times \frac{2\ell}{3} = \sqrt{\frac{4qE\ell}{3m}} = \sqrt{\frac{4pE}{3m}}$

7. $P_0A.\ell = PA(\ell - h)$

$\Rightarrow P_0\ell = \left(P_0 + \frac{2T}{r}\right)(\ell - h)$

$\Rightarrow \quad h = \frac{\ell}{\left(1 + \frac{P_0 r}{2T}\right)}$

8. From Snell's law

$1 \times \sin 90^\circ = n_{max} \sin 30^\circ$

$\Rightarrow n_{max} = 2$

9. For central bright fringe:

$\mu_w d \sin 30^\circ - (\mu - 1)t - d\sin\theta = 0$

$\Rightarrow \quad \sin\theta = -0.016$

$\tan\theta = \sin\theta = -0.016$

$\Rightarrow \quad \frac{y}{D} = -0.016 \Rightarrow$

$\Rightarrow y = -0.016 \text{ m} = 1.6 \text{ cm}$

10. Let the velocity of first fragment just after explosion be u in vertically downward direction. Then from conservation of momentum velocity of centre of mass of the other two fragment just after explosion will be u/2 in vertically upward direction. If the height of the object from ground when it explodes is h, then

$h = ut_1 + \frac{1}{2}gt_1^2$

$h = -\frac{u}{2}t_2 + \frac{1}{2}gt_2^2$

$\Rightarrow h = \frac{gt_1t_2(t_1 + 2t_2)}{2(2t_1 + t_2)}$

11. When the photoelectron emission, stops from the shell

$\frac{hC}{\lambda} = \frac{hC}{\lambda_0} + eV$

$\Rightarrow v_0 = \frac{hC}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = 12 \times 10^{-7}\left(\frac{1}{4 \times 10^{-7}} - \frac{1}{6 \times 10^{-7}}\right) = 1 \text{ Volt}$

The charge on capacitor $q = CV_0 = 2\ \mu C$

12. $\frac{hC}{\lambda} = \frac{hC}{\lambda_0} + e\frac{v_0}{2} + k_{max}$

$\Rightarrow k_{max} = \frac{ev_0}{2} = 0.5\,\text{eV}$

13. $z_1 = \sqrt{R_A^2 + X_L^2} = 10\ \Omega$

$z_2 = \sqrt{R_B^2 + x_C^2} = 10\,\Omega$

As $z_1 = z_2 \Rightarrow I_1 = I_2$

$\tan\phi_1 = -\frac{x_L}{R_A} = -\frac{6}{8} = -\frac{3}{4} \Rightarrow \phi_1 = -37°$

$\tan\phi_2 = \frac{x_C}{R_B} = \frac{8}{6} = \frac{4}{3} \Rightarrow \phi_2 = 53°$

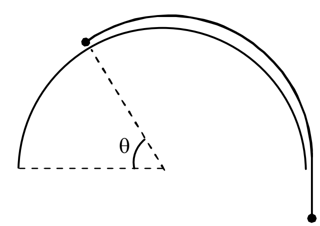
Phase difference between $I_1$and I_2 is 90°

$\Rightarrow I = \sqrt{I_1^2 + I_2^2} = 2I_1^2$

$\Rightarrow \quad I_1 = \frac{I}{\sqrt{2}} = \frac{14}{\sqrt{2}} = 7\sqrt{2}$ A

Power dissipated in the circuit = $I_1^2 R_A + I_2^2 R_B = 1372$ W

15-16. From conservation of mechanical energy. Loss in P.E. = Gain in K.E



$\Rightarrow \quad mg(a\theta - a\sin\theta) = 2\times\frac{1}{2}mv^2$

$\Rightarrow \quad V^2 = ag(\theta - \sin\theta)$

$mg\sin\theta - N = \frac{mv^2}{a}$

$\Rightarrow \quad N = mg(2\sin\theta - \theta)$

from equation (1)

$2V\frac{dv}{dt} = ag(1-\cos\theta)\frac{d\theta}{dt} = ag(1-\cos\theta)\frac{v}{a}$

$\Rightarrow \quad \frac{dv}{dt} = \frac{g}{2}(1-\cos\theta)$

19 In process B → C, VT^2 = constant

$\Rightarrow V\left(\frac{PV}{nR}\right)^2 = \text{constant}$

$\Rightarrow PV^{3/2} = $ constant

$C = C_V + \frac{R}{1-\alpha} = \frac{3R}{2} + \frac{R}{1-\frac{3}{2}} = -\frac{R}{2}$

So, in process B → C, temperature increases bet ΔQ is negative

Process C → A is adiabatic expansion